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No. 468

CHOICE OF PROFILE FOR THE WINGS OF AN AIRPLANE

By A. Toussaint and E. Carafoli

PART I

From L'Aeronautique, December, 1927

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 468.

CHOICE OF PROFILE FOR THE WINGS OF AN AIRPLANE.

By A. Toussaint and E. Carafoli.

The choice of the profile for the wings of an airplane is a problem which should be solved by a scientific method based on data obtained by systematic experimentation.

The problem, in its present form, may be stated as follows:

"To find a profile which has certain required aerodynamic characteristics and which encloses the spars, whose number, dimensions and separating distance are likewise determined by structural considerations."

These conditions, imposed in the choice of the profile, result from the specifications to be satisfied in the static tests. At present, the static test, corresponding to the case of accelerated flight at limited speed, requires the knowledge of the moment of the aerodynamic resultant at the angle of zero lift, and the possibility of controlling the magnitude of the corresponding absolute coefficient within more or less extensive limits. Let us add also that, according to practical constructional procedure, this absolute coefficient (C_{m_0}) should be very small.

*"Le choix du profile des ailes sustentatrices," from L'Aeronautique, December, 1927,

Moreover, a knowledge of the pressure distribution around the profile and along the wing span is necessary in all cases of static tests, since the diagrams of the load distribution of the static tests must be identical with the diagrams of the aerodynamic pressures.

Lastly, the profile should also have a profile drag as small as possible and good lifting qualities.

The search for a profile answering the various requirements can sometimes be made by consulting the collections of profiles investigated in the aerodynamic laboratories. At present, however, these data are generally insufficient, because they do not give the aerodynamic-pressure distribution and also because the investigated profiles have too large an absolute coefficient C_{m_0} , or dimensions and forms which do not satisfy the structural requirements.

In this case, the constructing engineer had, in the past, no other resource than that of adopting one or more profiles which he drew empirically and which he subsequently caused to be investigated in the aerodynamic laboratory. This long and burdensome process did not always yield the desired solution. Besides, the process had to be repeated very frequently without ever leading to general and conclusive results.

The only rational solution of the search for an airplane wing profile is found by scientific methods of drawing and cal-

culating so-called "theoretical" profiles. It is thus possible to find an infinite number of profile forms which answer all the proposed problems. For all these forms it is possible by starting from the data of the diagram to calculate the following essential characteristics:

1. The variation of the lift coefficient C_z , as a function of the angle of attack, and, particularly, the value of the angle of zero lift.

2. The variation of the moment coefficient C_{mA} , of the aerodynamic resultant with respect to the leading edge, and, particularly, the value C_{m0} of this moment for the angle of zero lift.

3. The aerodynamic pressure distribution along the wing chord for all angles of attack used in practice.

The theory for wings or cells of limited span allows the extension of these characteristics to all sections of the wing along the span.

Conversely, this method renders it possible to find the profiles answering the required conditions, which is generally the case in practice, as regards the value of the moment coefficient C_{m0} .

Convinced of the practical utility of these methods of drawing and calculating theoretical profiles, we have written quite

an exhaustive treatise on the subject.* At the request of the editor of this magazine, we have consented to give, with the exception of too greatly elaborated theoretical considerations, the essential elements for drawing theoretical profiles to answer actual problems. The same opportune question has been frequently put to us by engineers. We have had the satisfaction of learning from those interested, that the wing profiles, thus conceived and investigated, have verified the theoretical predictions in laboratory and flight tests.

Elementary Principles for Drawing a Theoretical Wing Profile

A theoretical profile is derived by conformal transformation from a circular contour.

In order to effect this conformal transformation of the circle into a profile a transformation function is used, which must satisfy certain mathematical conditions. The corresponding calculations are made by the use of complex variables.

Let the plane of the generating circle with center M be $O\xi\eta$ (Fig. 1). Each point P' , with coordinates ξ and η on this plane, will be represented by the complex variable:

$$\zeta = \xi + i\eta = \rho e^{i\theta}, \quad \text{with } \rho = OP' \quad (1)$$

*A. Toussaint and E. Carafoli, "Theorie et traces des profils d'ailes sustentatrices," in course of publication in the "Bulletin de la Chambre Syndicale des Industries aeronautiques, Vol. V, Bulletins Nos. 1-3, have already appeared.

Let us now consider the plane Oxy . Each point P , with coordinates x and y in this plane, will be represented by the complex variable:

$$z = x + iy = r e^{i\varphi}, \quad \text{with } r = OP. \quad (2)$$

The most general transformation function for accomplishing the conformal transformation of the circle in the plane ζ into a profile in the plane z can be expressed as

$$z = \zeta + \frac{x_1}{\zeta} + \frac{x_2}{\zeta^2} + \dots + \frac{x_n}{\zeta^n}. \quad (3)$$

The form and aerodynamic characteristics of the transformed profile depend on the number of terms of this function and on the parameters $x_1, x_2, x_3, \dots, x_n$ which, in the general case, are complex quantities. In particular, the first parameter x_1 may be written

$$x_1 = c^2 e^{i\alpha\gamma} \quad (4)$$

and the half-amplitude (demi-argument) γ , characterizes the direction of the second axis of the profile, that is, the direction of the relative wind for which the aerodynamic resultant passes through the center M of the generating circle, when the ζ and z planes are superposed and the axes $O\zeta\eta$ and Oxy coincide.

In practice, the parameters x_1, x_2, \dots, x_n are determined in the following manner.

By assuming the roots $(-\lambda), \lambda_1, \lambda_2, \dots, \lambda_n$ which nullify the derivatives $\frac{dz}{d\xi}$ of the transforming function, we obtain the equation

$$\begin{aligned} \frac{dz}{d\xi} &= 1 - \frac{x_1}{\xi^2} - 2 \frac{x_2}{\xi^3} - \dots - n \frac{x_n}{\xi^{n+1}} \\ &= \left(1 + \frac{\lambda}{\xi}\right) \left(1 - \frac{\lambda_1}{\xi}\right) \left(1 - \frac{\lambda_2}{\xi}\right) \dots \left(1 - \frac{\lambda_n}{\xi}\right) \end{aligned} \quad (5)$$

from which, by identification of the two developments, we obtain

$$\lambda_1 + \lambda_2 + \dots + \lambda_n = \sum_1^n \lambda_i = \lambda \quad (6)$$

$$x_1 = c^2 e^{2i\gamma} = \lambda^2 - \sum_1^n \lambda_i \lambda_j \quad (7)$$

$$\begin{cases} 2x_2 = -\lambda \sum_1^n \lambda_i \lambda_j + \sum_1^n \lambda_i \lambda_j \lambda_k \\ 3x_3 = \dots \end{cases} \quad (8)$$

The roots $(-\lambda), \lambda_1, \lambda_2, \dots, \lambda_n$, in the general case, are complex quantities. It is customary, however, to take the root $(-\lambda)$ along the axis $O \xi$ of the real quantities, its magnitude being arbitrarily chosen. This magnitude characterizes the profile dimension, that is, the scale of the drawing.

Moreover, the roots must satisfy only the following theoretical conditions:

1. The corresponding representative points L, L_1, L_2, \dots, L_n must be within the circle. However, in the case of an airfoil terminating in a sharp trailing edge (which is the general

case in actual practice), one of the roots and only one is located on the circle. (Incidentally, it may be remarked that it is possible to draw a profile without a sharp trailing edge or with only a more or less rounded edge.) It is customary to choose the real root $\xi = -\lambda$, so that the corresponding representative point L is coincident with the point B' of the generating circle (Fig. 1).

2. The geometric resultant of the vectors OL_1, OL_2, \dots, OL_n corresponding to the roots $\lambda_1, \lambda_2, \dots, \lambda_n$ must be equal to $OL' = \lambda$ by virtue of equation (6).

These theoretical restrictions of the choice of roots are supplemented by lessons taught us by the practice of drawing. Thus all roots such as L_1 (Fig. 2) near the circle at A' will produce a slightly rounded point at A on the profile. Similarly, a root such as L_j will produce a boss at J on the profile.

Practical Method for Drawing a Theoretical Wing Profile with a Sharp Trailing Edge

The practical method for drawing a theoretical wing profile with a sharp trailing edge, according to the preceding general principles, comprises the following operations:

1. Draw the coordinate axes for the planes ξ and z in coincidence, and let $Ox\xi$ and $Oy\eta$ be these axes (Fig. 3).

2. Choose the characteristic magnitude λ according to the dimensions of the profile to be obtained, knowing that the chord l of the profile will be approximately 4λ .

3. Lay off $OB' = \lambda$ on the $O-x, \xi$ axis in the negative direction. The point B' is located on the generating circle and represents the real root $\xi = -\lambda$.

4. Choose the other roots $\lambda_1, \lambda_2, \dots, \lambda_n$ (equal in number to the terms desired for the transformation function) so as to satisfy the abovementioned conditions.

The center M of the generating circle is not yet known, but it is, in practice, not far from the origin O , so that the condition relative to the position in the circle for the representative points L_1, L_2, \dots, L_n can be realized with an approximation, which in general is proved sufficient by the continuation of the drawing. The parameters $x_1, x_2, x_3, \dots, x_n$ are then calculated by means of equation (5).

5. Draw the second axis of the profile. It is known that the amplitude of the second axis is equal to half the amplitude of the parameter:

$$x_1 = c^2 e^{i2\gamma} = \lambda^2 - \sum_1^n \lambda_i \lambda_j \quad (7)$$

To that effect, we determine the point O' representative of

$$-\frac{x_1}{\lambda} = -\lambda + \frac{1}{\lambda} \sum_1^n \lambda_i \lambda_j = -\frac{c^2}{\lambda} e^{i2\gamma} \quad (7')$$

Since $OB' = -\lambda$, it will suffice to draw through the point B' a straight line $B'C'$ making with Ox an angle equal to the amplitude of the complex quantity $\sum_{i=1}^n \lambda_i \lambda_j$. On this axis and with its sign we lay off a length $B'C'$ such that

$$\overline{B'C'} = \frac{\text{mod } \sum_{i=1}^n \lambda_i \lambda_j}{\lambda} = \frac{\left| \sum_{i=1}^n \lambda_i \lambda_j \right|}{\lambda}.$$

We then have

$$c^2 = x_1 = \lambda \times OC', \quad (9)$$

and the second axis is the bisector of the angle

$$B' OC' = 2\gamma.$$

The quantity $c^2 = x_1$, which appears in the expression for the moment of the resultant, can then be calculated by measuring OC' directly on the diagram, or by solving the triangle $B' OC'$.

6. Draw the first profile axis for the purpose of obtaining a fixed moment coefficient C_{m_0} .

The direction of the first axis corresponds to that of zero lift.

We shall see farther on, in connection with the aerodynamic characteristics of the profile, that the value of C_{m_0} is given by the expression

$$C_{m_0} \approx 8 \pi \frac{c^2}{l^2} (\beta - \gamma) \quad (10)$$

in which l is the profile chord, the value of which is approximately 4λ , or better still $2(\lambda + \overline{OC'})$, and β is the amplitude of the first axis or axis of zero lift.

For the required value of C_{m_0} , we therefore have

$$\beta - \gamma \approx C_{m_0} \frac{4(\lambda + \overline{OC'})^2}{8\pi c^2} = C_{m_0} \frac{(\lambda + \overline{OC'})^2}{2\pi \lambda \overline{OC'}} \quad (11)$$

Following laboratory practice, we consider C_{m_0} positive, when it tends to diminish the angle of attack. Thus, for $C_{m_0} > 0$, we have $\beta > \gamma$; for $C_{m_0} = 0$, we have $\beta = \gamma$; and lastly, for $C_{m_0} < 0$, we have $\beta < \gamma$.

The first profile axis is the straight line $B'MI$ drawn through the point B' and making with the axis Ox, ξ the angle β determined by formula (11).

7. Draw the generating circle. The center M of the generating circle is found on the first axis, which meets the axis Oy, η at the point M_0 . The distance M_0M is arbitrarily chosen very small. (This distance characterizes, in part, the relative maximum profile thickness.) One may thus define the center M by adopting an arbitrary value, a little larger than unity, for the ratio

$$k = \frac{B'M}{\lambda} = \frac{a}{\lambda}$$

$a = \overline{B'M}$ is the radius of the generating circle.

Lastly, we may characterize the position of the center M

by the amplitude δ of the direction OM. From the diagram (or by calculation of triangle B'OM), the value of the modulus $|\mu| = \overline{OM}$.

The generating circle is drawn through the point B' on the axis O-x, ξ and we may verify the required condition for the interior position of the points L_1, L_2, \dots, L_n .

8. Draw the auxiliary circle corresponding to the term $\frac{x_1}{\xi}$ of the transformation function.

In drawing the profile and the generating circle on the axes Ox' and Oy' so that Ox' coincides with the second axis, the position of any point P' on the circle and of the corresponding point P on the profile are characterized by the complex variables ζ' and z' respectively, so that we have

$$z = z' e^{i\gamma} \quad \text{and} \quad \zeta = \zeta' e^{i\gamma}$$

from which we derive

$$z' = \zeta' + \frac{c^2}{\zeta'^2} + \frac{x_2 e^{-2i\gamma}}{\zeta'^2} + \frac{x_3 e^{-4i\gamma}}{\zeta'^3} + \dots \quad (12)$$

In this form, it is obvious that, in order to find a point P of the profile, it is first necessary to evaluate the sum

$$\zeta' + \frac{c^2}{\zeta'^2}$$

It has been demonstrated that the auxiliary transformation

$$z'_{11} = \frac{c^2}{\zeta'}$$

leads to another circle (called auxiliary circle) defined as follows. The center M_1 of this circle is located on the line OM_1 symmetrical to OM with respect to the new axis oy' at a distance OM_1 calculable by the formula

$$\overline{OM_1} = |\mu_1| = \frac{c^2}{a^2 - |\mu|^2} |\mu| \quad (13)$$

Lastly, the radius a_1 of this circle is given by the equation

$$a_1 = \frac{c^2}{a^2 - |\mu|^2} a \quad (14)$$

It can be shown finally that the auxiliary circle passes through the point C' , so that

$$a_1 = \overline{M_1 C'}$$

9. In order to accomplish the partial transformation corresponding to the first two terms of the transformation function, it is sufficient to apply to the circles M and M_1 the construction of Trefftz, commonly applied in the case of Joukowski profiles, or profiles with two terms.

For the sake of completeness, we shall briefly review said construction.

With the origin O as center, (Fig. 4) a circle is drawn which is divided into equal arcs (generally of 30° or 15°) starting from the second axis Ox' . The radius vectors, corresponding to these arcs, define, on the circle M , the points

P'_0, P'_1, \dots, P'_k , and, on the circle M_1 , the points Q'_0, Q'_1, \dots, Q'_k , which are conjugated two by two for amplitudes of equal magnitude but of opposite signs.

The parallelogram constructed upon two conjugate radius vectors, OP'_k and OQ'_k defines a point P_k of the desired transformation. The construction on the axes Ox' and Oy' leads to the addition or subtraction of the conjugate radius vectors.

For example, $OP_0 = OP'_0 + OQ'_0$ and $OP_3 = OP'_3 - OQ'_3$. We thus obtain just as many points P_k as we wish, according to the magnitude of the adopted angular equidistance.

10. To make the partial transformations corresponding to the other terms of the transformation function. The other terms of the transformation function have the general form

$$\frac{x_n e^{-i(n+1)\gamma}}{\zeta'^n},$$

according to the expression 12 in the system of coordinate axes Ox' and Oy' . According to equation (5), the parameter x_n can be calculated in terms of the roots of $\frac{dz}{d\zeta}$ and may be written

$$x_n = c_n e^{i\sigma n}$$

Every point P'_k of the generating circle is defined likewise, with respect to the axes Ox' and Oy' , by the expression

$$\zeta' = \rho_k e^{i\theta_k}$$

from which it follows that the amplitude φ_n of the term is

$$\varphi_n = \sigma_n - (n+1)\gamma - n\theta_k \quad (15)$$

and the modulus is

$$\left| \frac{x_n e^{-i(n+1)\gamma}}{\zeta'^n} \right| = \frac{c_n}{\rho_k^n} = \frac{|x_n|}{\rho_k^n} \quad (16)$$

We can calculate φ_n and $\frac{x_n}{\rho_k^n}$ for the various points P'_0, P'_1, \dots, P'_k , previously determined according to the adopted angular equidistance.

The partial transformation corresponding to one of the other terms of the function will then consist of a positional correction, the amplitude of which, with respect to the axis Ox' is φ_n , or $(\varphi_n + \gamma)$ with respect to the axis Ox , the magnitude and sign of which are given by

$$\frac{|x_n|}{\rho_k^n}$$

Thus, for all points P_k , corresponding to the first two terms of the transformation, the third term

$$\frac{x_2 e^{-3i\gamma}}{\zeta'^2}$$

defines a positional correction $P_k P_k$ (Fig. 5), the amplitude of which is

$$\varphi_2 = \sigma_2 - 3\gamma - 2\theta_k$$

with respect to Ox' , or

$$\varphi_2 + \gamma = \sigma_2 - 2\gamma - 2\theta_k$$

with respect to Ox .

σ_2 being the amplitude of x_2 calculable by equation (8), and θ_k being equal to k times the adopted angular equidistance, since it concerns a point P_k , the antecedent of which is P'_k on the generating circle, the magnitude or modulus of the correction is

$$\frac{x_2}{\rho_k^2} = \frac{x_2}{(OP_k)^2}.$$

Remarks.— In practice, we compute the value of $\frac{|x_n|}{\rho_k^n}$ for a few points P'_0, P'_1, \dots, P'_k and draw a curve of the variations of this quantity, which is generally small. The other values are then derived by interpolation, according to the graph obtained.

We can therefore easily make the various positional corrections corresponding to the partial transformations due to the terms of the function other than the first two.

Remarks on the Practical Choice of the Number of Terms of the Transformation Function.

In practice, it is not necessary to resort to a very complicated transformation function. One of three terms, such as

$$z = \zeta + \frac{x_1}{\zeta} + \frac{x_2}{\zeta^2} \quad (32)$$

is capable of furnishing an infinite number of profile forms with the required C_{m_0} . In this case, the drawing will reduce itself to the graphic construction for the first two terms

$$\left(\zeta' + \frac{c^2}{\zeta'}\right)$$

and to a single positional correction due to the term

$$\frac{x_2 e^{-i\gamma}}{\zeta'^2}$$

However, experience shows that, in this case, the position corrections are very important. It follows that the envelope of the points corresponding to the first two terms $\left(\zeta' + \frac{c^2}{\zeta'}\right)$ is very different from the form of the final profile. It is important, from the practical point of view, after the first construction $\left(\zeta' + \frac{c^2}{\zeta'}\right)$, to be sure of the form and approximate proportions of the final profile. We thus find, without drawings or useless computations, that the profile sought will effectively and very closely envelop the spars necessary for the construction.

Under these conditions, we are frequently led to use a transformation function of three terms corresponding to a particular case of a four-term function. This particular case consists in nullifying the third term in x_2 . The function is then written

$$z = \zeta + \frac{x_1}{\zeta} + 0 + \frac{x_3}{\zeta^3} \quad (33)$$

In order to obtain this particular four-term function, it suffices to choose the roots λ_1 , λ_2 and λ_3 in such a manner that $x_2 = 0$.

We have already found that

$$2x_2 = -\lambda \sum_1^n \lambda_i \lambda_j + \sum_1^n \lambda_i \lambda_j \lambda_k \quad (8)$$

When $n = 3$, the annullment of x_2 leads to the condition

$$\lambda \sum_1^3 \lambda_i \lambda_j = \sum_1^3 \lambda_i \lambda_j \lambda_k,$$

that is to say,

$$\lambda (\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3) = \lambda_1 \lambda_2 \lambda_3.$$

This condition is verified for

$$\lambda_1 = \lambda \quad \text{with} \quad \lambda_2 = -\lambda_3 \quad (34)$$

The roots are therefore so chosen that the corresponding representative points satisfy the conditions

$$\overline{OL}_1 = \lambda \text{ directed along } Ox$$

and
$$\overline{OL}_2 = -\overline{OL}_3 \text{ in any direction (Fig. 6).}$$

By this particular choice of roots, drawing the profile is rendered very simple by a graphical construction $(\zeta' + \frac{c^2}{\zeta'})$ and by a positional correction, generally very small and corresponding to the term

$$\frac{x_3 e^{-i4\gamma}}{\zeta'^3}.$$

Remarks.— In practice, we can do without defining the roots $\lambda_2 = -\lambda_3$. If we assume an arbitrary value for the segment $B'C'$ (Fig. 7), the parameters x_1 and x_3 of function 33 are determined. In fact, we have

$$x_1 = \lambda \overline{OC'}$$

On the other hand,

$$\overline{B'C'} = \frac{\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3}{\lambda} = -\frac{\lambda_2^2}{\lambda}.$$

Since

$$3x_3 = \lambda \lambda_1 \lambda_2 \lambda_3 = -\lambda^2 \lambda_2^2,$$

we obtain

$$x_3 = +\frac{1}{3} \lambda^2 \lambda \overline{B'C'} = \frac{\lambda^3}{3} \overline{B'C'}.$$

The above exposition corresponds to the method of Von Mises. Later we shall give a simpler and more general construction, which will enable us to predict the evolution of the profile form.

Example of a Profile Drawing

with C_{m0} Fixed in Advance

We shall give (Fig. 8) the example of drawing a theoretical profile for which the condition

$$C_{m0} = 0.055$$

is required. We shall select for this drawing the three-term function

$$z = \zeta + \frac{x_1}{\zeta} + 0 + \frac{x_2}{\zeta^3} \quad (33)$$

Following the order previously indicated, we shall have to perform the following operations:

- 1st. Drawing the coordinate axes Ox and Oy or $O\xi$ and $O\eta$.
- 2d. Choosing λ .-- We shall take $\lambda = 6$ cm, in order to obtain a profile chord about 2λ cm long.
- 3d. We lay out $OB' = -\lambda = -6$ cm on the negative direction of Ox .
- 4th. Choosing the roots and computing x_1 and x_3 .
According to what has preceded, we shall have

$$\lambda_1 = \lambda = 6 \text{ cm}$$

and we assume that

$$\lambda_2 = -\lambda_3 = 0.345 \lambda e^{i55^\circ} = 2.075 e^{i55^\circ}$$

The representative points L_2 and L_3 will be on the line of amplitude 55° at the distances

$$\overline{OL_2} = -\overline{OL_3} = 20.75 \text{ cm.}$$

From this choice of the roots, we obtain the following equation

$$\begin{aligned} \frac{dz}{d\xi} &= 1 - \frac{x_1}{\xi^2} - 3 \frac{x_3}{\xi^4} = \left(1 + \frac{\lambda}{\xi}\right) \left(1 - \frac{\lambda_1}{\xi}\right) \left(1 - \frac{\lambda_2}{\xi}\right) \left(1 - \frac{\lambda_3}{\xi}\right) \\ &= 1 - \frac{\lambda^2}{\xi^2} - \frac{0.119 \lambda^2 e^{i110^\circ}}{\xi^2} + \frac{0.119 \lambda^4 e^{i110^\circ}}{\xi^4}, \end{aligned}$$

from which we deduce

$$x_1 = \lambda^2 (1 + 0.119 e^{i110^\circ}) = 36 (1 + 0.119 e^{i110^\circ})$$

$$x_2 = 0$$

$$x_3 = - \frac{0.119 \lambda^4 e^{i110^\circ}}{3} = - 51.6 e^{i110^\circ}$$

5th. Drawing the second axis. We construct

$$- \frac{x_1}{\lambda} = - 6 - 0.716 e^{i110^\circ}$$

In the direction of amplitude $+110^\circ$ we lay out

$$\overline{B'C'} = - 0.716 \text{ cm, thus finding}$$

$$\gamma = \frac{B'OC'}{2} = \frac{6.6^\circ}{2} = 3.3^\circ,$$

from which we get the second axis, passing through the origin O.

6th. Drawing the first axis. The amplitude β of this first axis will be determined by the condition relative to $C_{m_0} = 0.055$. We lay out on the drawing, $\overline{OC'} = 5.77 \text{ cm}$, and calculate

$$\beta - \gamma = 0.055 \times \frac{(6 + 5.17)^2 \times 57.3}{6.28 \times 6 \times 5.77} = 2^\circ.$$

We therefore have $\beta = \gamma + 2^\circ = 5.3^\circ$, whence the first axis of amplitude 5.3° passing through B' .

7th. Drawing the generating circle. The center M' is chosen in the direction OM with amplitude $\delta = 52^\circ + \gamma$, and we lay out on the drawing

$$\overline{OM} = |\mu| = 0.73 \text{ cm,}$$

$$\overline{B'M} = a = 6.45 \text{ "}$$

$$C^2 = \lambda OC' = 34.6 \text{ cm.}$$

8th. Drawing the auxiliary circle. Draw the line OM_1 symmetrically with OM with respect to Oy' (perpendicular to the second axis). Then compute

$$\overline{OM}_1 = |\mu_1| = \frac{34.6}{(6.45)^2 - 0.73} \times 0.73 = 0.618 \text{ cm}$$

from which we obtain the circle with the center M_1 and passing through C' .

9th. Partial transformation corresponding to the first two terms. Having adopted the angular equidistance of 15° we proceed with the graphic construction on the conjugate points of the circles M and M_1 .

10th. Positional corrections corresponding to the term $\frac{x_3}{\xi^3}$. By computing the amplitude and the correction modulus for each of the points $1', 2', 3', \dots$ of the generating circle, we obtain

$$(\varphi_3 + \gamma) = 110^\circ - 3 \times 3.3^\circ - 3 \times n\theta$$

with respect to the direction Ox , and

$$\frac{|x_3|}{\rho_n^3} = - \frac{51.6}{\rho_n^3}$$

in which 110° is the amplitude of x_3 and $n\theta = n \times 15^\circ$ is the amplitude of the point n' on the generating circle.

Through each point resulting from the graphic construction on the circles M and M_1 the direction of amplitude $(\varphi_3 + \gamma)$ is drawn, and the length $\frac{|x_3|}{\rho_n^3}$ is laid out in the negative direc-

tion, since the corrections have the minus sign. As easily seem, the positional corrections are relatively small, so that the final profile differs very little from the envelope of the points resulting from the first two terms.

Figures 9 and 10 give two other examples of drawing theoretical profiles. Figure 6 corresponds to the case of a three-term function of the form 33, while Figure 7 corresponds to the case of a three-term function of the form 32. In the latter case, the positional corrections are important and the envelope of the points, corresponding to the partial transformation due to the first two terms, furnishes no information whatever regarding the form of the final profile.

To be followed by Technical Memorandum No. 469, containing the translation of the remainder of this article.

Translation by
National Advisory Committee
for Aeronautics.

Fig.1

Fig. 2

Fig. 3

Fig. 7

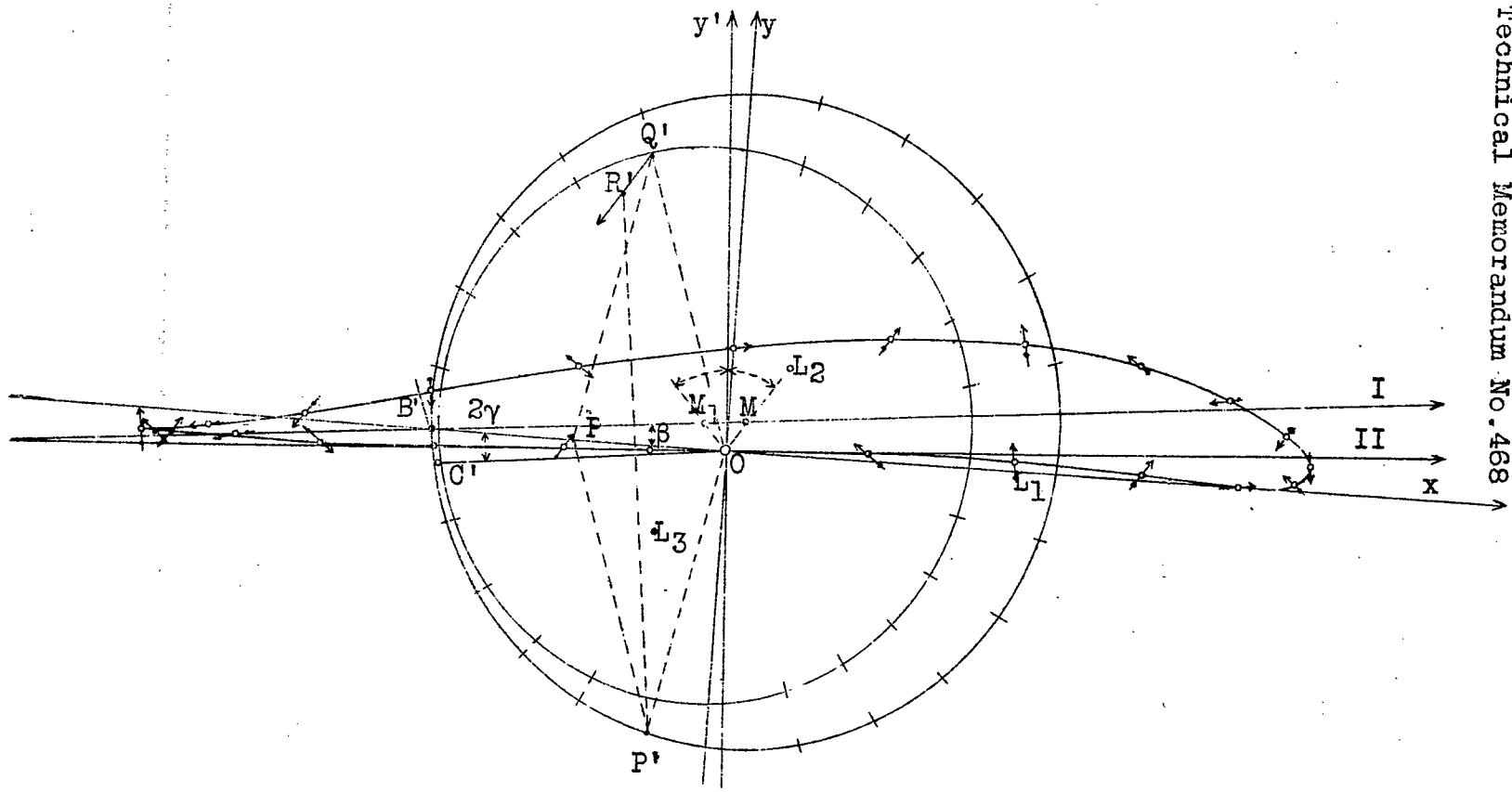


Fig. 8

Fig. 8

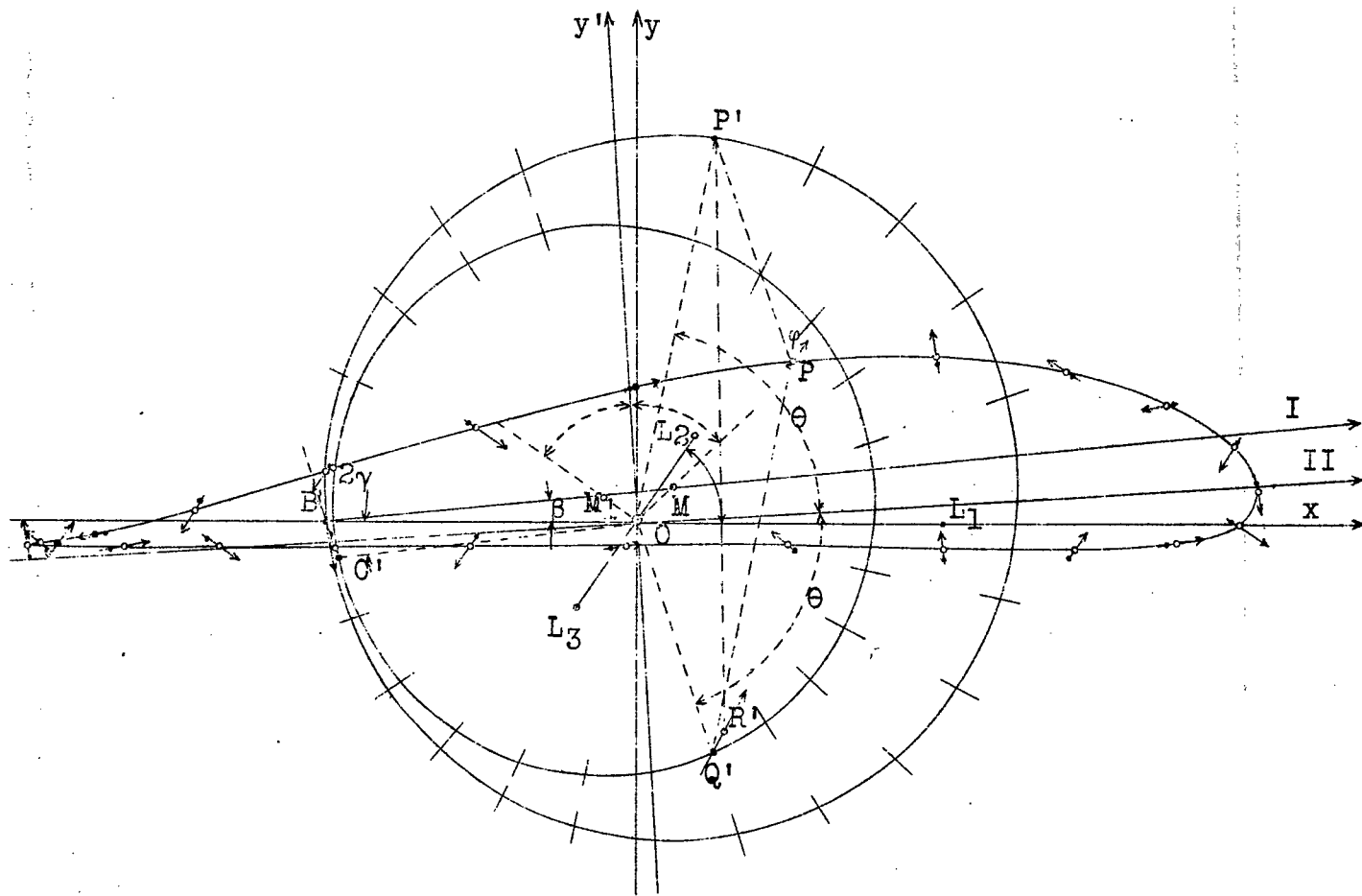
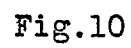


Fig. 9



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